رياضيات هندسية (1)

Second

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	التخصص والمستوى :	السلطة الوطنية الفلسطينية
Linear and diffe	إسم المساق :	جامعة فلسطين التقنية طولكرم"خضوري"
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الشعبة:

Q1. Mark each statement by true or false. (25 points.)

1) (T)If S={x_1,x_2,x_3} are linearly independent vectors in R^3 ,then S form a basis for R^3 .

~~T~~ 2) (F) An $n \times n$ matrix A is singular if the nullity (A)=0.

3) (F) If L :V$\rightarrow$W is a linear transformation, then kernel (L) is a subspace of the range of (L).

F 4) (T) If A is An $n \times n$ matrix and $\lambda_1,......,\lambda_n$ are the eigenvalues of A with corresponding eigenvector $X_1,..........X_n$ respectly ,and if $X_1,..........X_n$ are linearly independent then $\lambda_1,......,\lambda_n$ are distinct .

5)(F)The product of the eigenvalues for An $n \times n$ matrix (A) is equal to trace of(A) .

6)(T) If rank [A|b] =rank(A) then the system Ax=b is consistent.

7)(F)The function f(x , y)= $\sqrt{x^2-y^2}$ is continuous in the entire plane.

8)(T)A basis for a vector space V is the smallest spanning and the largest linearly independent subset of V.

9)(T) An $n \times n$ matrix A is singular iff one of it eigenvalues is zero .

F 10)(T)If A is a 3×4 matrix, then the row vector of A are linearly dependent

تم الرفع بواسطة م. معن أبو عيسى

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F_{yx}

Q2) a. $z=\dfrac{xy+x\cos y}{x^2y}$ find $\dfrac{\partial z}{\partial x \partial y}\Big|_{(x,y)=(1,\frac{\pi}{2})}$ (10 points)

خلف الورقة

~~$F_y = (xy + x\cos y)$~~

$F_y = \dfrac{xy + x\cancel{\cos y}}{x^2y} \Rightarrow F_y = \dfrac{(xy) * x^2 - x^2y(x)}{(x^2y)^2}$

$= \dfrac{x^2yx^2 - x^3yx}{x^4y^2} = \dfrac{x(x^3y - x^3y)}{x(x^3y^2)}$

$\Rightarrow \cancel{F_y} = \dfrac{x^3y - x^3y}{x^3y^2} =$

$\Rightarrow f_y =$

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$$z = \frac{xy + x\cos y}{x^2 y}$$ find $\frac{\partial z}{\partial x \partial y}$ & $x=1$, $y=\frac{\pi}{2}$

$$F_y = \frac{x^2y * x - xy * 2xy}{(x^2y)^2} = \frac{x^3y - 2x^2y^2}{x^4y^2}$$

(1)

$$\frac{\cancel{x^2}(xy - 2y^2)}{\cancel{x^2}(x^2y^2)}$$

$$\Rightarrow F_y = \frac{xy - 2y^2}{x^2y^2}$$

$$\Rightarrow F_y = \frac{\cancel{y}(x-2y)}{\cancel{y}(x^2y)} \Rightarrow \boxed{F_y = \frac{x-2y}{x^2y}}$$

(2)

$F_{y\cancel{x}} =$

$$F_y = \frac{x-2y}{x^2y}$$

$$F_{yx} = \frac{(x^2y * 1) - (x-2y) * 2xy}{(x^2y)^2}$$

$$\Rightarrow F_{yx} = \frac{x^2y - (2x^2y - 4xy^2)}{(x^4y^2)}$$

$$\Rightarrow F_{yx} = \frac{xy(x - (2x - 4y))}{\cancel{x}y(x^3y)}$$

$$F_{yx} = \frac{x - 2x + 4y}{x^3 y}$$

$$= \frac{1 - 2(1) + 4\left(\frac{\pi}{2}\right)}{1 * \frac{\pi}{2}}$$

$$= \frac{-1 + 2\pi}{\frac{\pi}{2}} \quad \boxed{= \frac{-2 + 4\pi}{\pi}}$$

b. 1) Show that $L: R^3 \to R^3$ defined by $L(\vec{x}) = (x_1, 0, x_1 + x_2)$ is a linear transformation. (5 points)

$\alpha L(x) + \beta L(y) = \alpha(x_1, 0, x_1+x_2) + \beta(y_1, 0, y_1+y_2)$

$= (\alpha x_1, 0, \alpha x_1 + \alpha x_2) + \beta y_1, 0, \beta y_1 + \beta y_2)$

$= \alpha L x + \beta L y \to$ transformation

2. Find ker(L). (5 points)

$x_1, 0, x_1+x_2 = (0, 0, 0)$

$\begin{pmatrix} x_1 \\ 0 \\ x_1+x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = x_1 = -x_2 \Rightarrow$ Ker L spaning by $\begin{pmatrix} e_1 \\ 0 \\ -e_3 \end{pmatrix} =$

3. Let S be the subspace of R^3 spanned by $\{e_2, e_3\}$. Find $L(S)$. (5 points)

$\begin{pmatrix} 0 \\ e_2 \\ e_3 \end{pmatrix}$

Q3. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 5 & 4 \\ 1 & 2 & 4 & -1 \end{bmatrix}$. Find

1. A basis for the row space of A: (10 points)

$\Rightarrow \begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & -1 & -1 & 2 \\ 0 & 1 & 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 3 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

A basis for the row space of A is

row $(1\ 1\ 3\ 1)$ $\Rightarrow$ $(1\ 1\ 3\ 1)$

row $(0\ 1\ 1\ -2)$ $\Rightarrow$ ~~$(2\ 1\ 5\ 4)$~~

2. A basis for the column space of A: (5 points)

A basis for column space of A

column that has 1 only 1 & 2

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

3. A basis for the null space of A : (10 points)

basis for the null space of A

$\left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 2 & 1 & 5 & 4 & 0 \\ 1 & 2 & 4 & -1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array}\right]$

$\boxed{\text{Page 4}}$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

التكملة خلف الورقة.

x_4 is free variable $x_4 = \beta$, $x_3 = \alpha$

$x_2 = -x_3 + 2x_4 = -\alpha + 2\beta$

$x_1 = -x_2 - 3x_3 - x_4 = -(-\alpha + 2\beta) - 3\alpha - \beta = \alpha + 2\beta$ ~~$3\alpha + \beta$~~

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$$x_1 = \alpha + 2\beta - 3\alpha - \beta$$

$$x_2 = -\alpha + 2\beta$$

$$X = \begin{pmatrix} \alpha + 2\beta - 3\alpha - \beta \\ -\alpha + 2\beta \\ \alpha \\ \beta \end{pmatrix}$$

$$\boxed{= \alpha \begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}}$$

الجواب

4. a . Find the eigenvalues and the corresponding eigenspaces for the matrix

$$A=\begin{bmatrix}1&1&1\\0&2&1\\0&0&1\end{bmatrix}$$, Is A diagonzlizable? Why?

b. Find A^{100}. **Do NOT compute X^{-1}**

$$A = \left|\begin{matrix}1-\lambda & 1 & 1\\ 0 & 2-\lambda & 1\\ 0 & 0 & 1-\lambda\end{matrix}\right| = 0$$

$$1-\lambda\left((2-\lambda)(1-\lambda) - (0) + 1(0)\right)$$

$$\Rightarrow 1-\lambda\left(2-2\lambda-\lambda+\lambda^2\right)$$

$$= 1-\lambda\left(2-3\lambda+\lambda^2\right)$$

$$1-\lambda\left(\lambda^2-3\lambda+2\right) =$$

$$\lambda^2-3\lambda+2-\lambda^3+3\lambda^2-2\lambda$$

$$-\lambda^3+4\lambda^2-5\lambda+2$$

$$\lambda^3-4\lambda^2+5\lambda-2$$

$$1-4+5-2$$

$$(\lambda-1)(\lambda-2)$$

$$\lambda=1 \quad \lambda=2$$

$$1-\lambda=0 \Rightarrow \lambda=1$$

$$\therefore \lambda_1=1, \lambda_2=1, \lambda_3=2$$

For $\lambda=2$

$$\left[\begin{array}{ccc|c}-1&1&1&0\\0&0&1&0\\0&0&-1&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&-1&-1&0\\0&0&1&0\\0&0&1&0\end{array}\right] \Rightarrow$$

$$\left[\begin{array}{ccc|c}-1&1&1&0\\0&0&1&0\\0&0&-1&0\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&-1&-1&0\\0&-1&0&0\\0&0&-1&0\end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c}1&-1&-1&0\\-\phi&1&0&0\\0&0&-1&0\end{array}\right]$$

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for $\lambda = 1$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

~~A not~~

A not diagonzlizable because ~~[illegible]~~ $= 0$

(b) find A^{100} $\boxed{\cancel{= X A^{100} X^{-1}}}$

$$A^{100} = X \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{100} X^{-1}$$

when it diagonlizable

D

$$A^{100} = X D^{100} X^{-1}$$